Indian Statistical Institute, Bangalore Centre.
Back-paper Exam : Graph Theory

Instructor : Yogeshwaran D.

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Max. points : 40. **Time Limit : 3 hours.**

Answer any four questions only.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class, mention it clearly. See the end of the question paper for notations.

1. Let G be a finite, simple, undirected graph and $s \neq t \in V(G)$. State and prove Menger's theorem for edge disjoint paths from s to t in G. (**10**)

2. (a) A tree T has a perfect matching iff $o(T - v) = 1$ for all $v \in T$. (**5**)

   (b) Show that $2\alpha'(G) = \min_{S \subset V}\{|V(G)| - d(S)\}$ where $d(S) = o(G - S) - |S|$. (**5**)

3. Compute the eigenvalues of the Laplacian and Adjacency matrices of the the cycle graph C_n. (**10**)

4. Find the chromatic polynomial of the cycle graph C_n, wheel graph W_n (i.e., the graph obtained by adding a new vertex to C_n and connecting it to all the n vertices of C_n) and the path graph P_n. (**10**)

5. Consider a directed graph $G = (V, E)$ and $s \neq t \in V$. Further assume that there are no incoming edges at s or no out-going edges at t. If f is an integral flow of strength k, show that there exist k directed paths $p_1, \ldots, p_k$ such that for all $e \in E$, $|\{p_i : e \in p_i\}| \leq f(e)$. (**10**)

Some notations :

- Unless mentioned, G is assumed to be a finite, undirected, simple graph everywhere.

- $o(G)$ - Number of odd components in a graph G.
- $\alpha'(G)$ - Maximum independent edge set.